Exercises Module 8
Correction

Exercise 8.1

The rate of oxygen transfer is given by:

$$A\, n_{O_2,0} = -\frac{d}{dt}\left(c_{O_2,G} * \frac{4}{3}\pi r^3\right)$$

$$4\pi r^2\, k_{c,L}\left(c_{O_2,L,0} - c_{O_2,L,bulk}\right) = -\frac{d}{dt}\left(c_{O_2,G} * \frac{4}{3}\pi r^3\right)$$

Simplify (take out constants and cancel) and separate, remember that r is a function of time and the concentrations are not….

$$-\frac{3\ k_{c,L}\left(c_{O_2,L,0} - c_{O_2,L,bulk}\right)}{c_{O_2,G}} r^2 = \frac{dr^3}{dt}$$

Here we have to apply the chain rule to show that $\frac{dr^3}{dt} = 3r^2 \frac{dr}{dt}$ when r(t). Therefore

$$-\frac{k_{c,L}\left(c_{O_2,L,0} - c_{O_2,L,bulk}\right)}{c_{O_2,G}} = \frac{dr}{dt}$$

Separate, integrate and apply initial condition (i.e. at t = 0 , r = 0.05 cm)

$$r = 0.05\ cm - \frac{k_{c,L}\left(c_{O_2,L,0} - c_{O_2,L,bulk}\right)}{c_{O_2,G}}\ t$$

Since $c_{O_2,L,bulk}$ = 0, $c_{O_2,L,0}$ = $1.5 \cdot 10^{-3}$ M, and $c_{O_2,G}$ = 0.0404 M (assuming 1 bar and 25°C, using ideal gas law) then:

$$r = 0.05\ cm - \frac{1.5 \cdot 10^{-3}\ M\ k_{c,L}}{0.0404\ M}\ t$$

Or

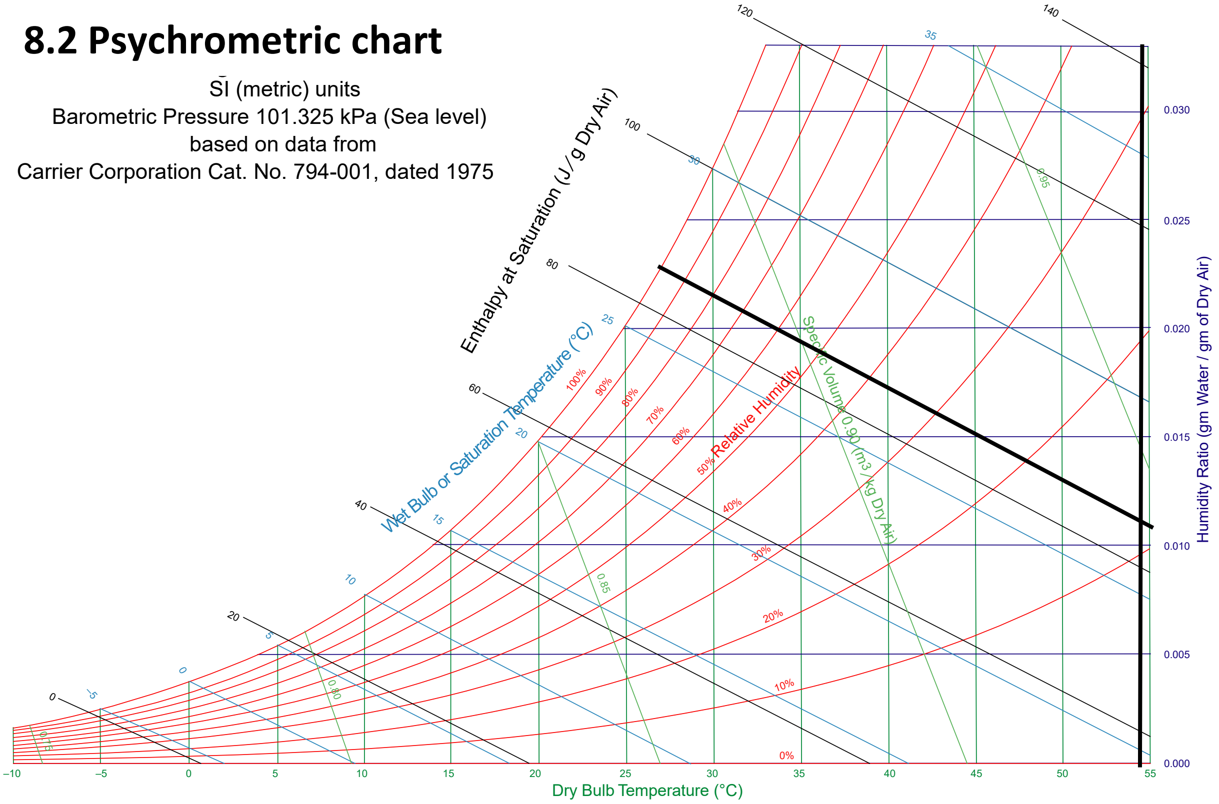
$$r = 0.05\ cm - 0.0371\ \ k_{c,L}\ t$$

Knowing that after 7 min (420 sec) the bubble radius is 0.027 cm then:

$$0.027\ cm = 0.05\ cm - (0.0371)\ \ k_{c,L}\ (420\ s)$$

And then $k_{c,L} = 1.48\ 10^{-3}\ cm\ s^{-1}$

Exercise 8.2



From the chart (intersection of thick black lines) the humid ratio is about 0.0115 g water / g dry air

From slide 17 we see that $H_R = \frac{m_A}{m_G} = 0.6220\frac{x_{A,G}}{1-x_{A,G}}\, g\, water/g\, dry\, air$

Then $0.0115 = 0.6220\frac{x_{A,G}}{1-x_{A,G}}$ or $x_{A,G} = 0.0182$

Exercise 8.3

We are asked to find and expression for n_A through the membrane and can use the relation $n_A = K_c(c_{A,B,\,bulk} - c^{eq}_{A,B})$.

We can write expressions for the transfer at each side of the membrane.

$n_A = k_{c,B}(c_{A,B,\,bulk} - c_{A,B,0})$, and $n_A = k_{c,C}(c_{A,C,L} - c_{A,C,bulk})$.

From equation 4.16 module 4 we can also describe the transport in the solid membrane at steady state:

$$0 = \mathcal{D}_{\mathrm{AD}} \boldsymbol{\nabla}^2 \mathrm{c}_{A,D}$$

$$0 = \frac{d^2 \mathrm{c}_{A,D}}{dz^2}$$

With $\mathrm{c}_{A,D}(0) = \frac{c_{A,B,0}}{m_B}$ and $\mathrm{c}_{A,D}(L) = \frac{c_{A,C,L}}{m_C}$

Then $\mathrm{c}_{A,D} = C_1 z + C_2$ or after applying boundary conditions:

$\mathrm{c}_{A,D} = \left(\frac{c_{A,C,L}}{m_C \cdot L} - \frac{c_{A,B,0}}{m_B \cdot L}\right) z + \frac{c_{A,B,0}}{m_B}$

Using Fick's law we can write an expression for the diffusive flux through the membrane:

$\mathrm{j}_{\mathrm{A,z}} = -\mathcal{D}_{\mathrm{AD}} \frac{d\mathrm{c}_{A,D}}{dz} = \mathcal{D}_{\mathrm{AD}} \left(\frac{c_{A,B,0}}{m_B \cdot L} - \frac{c_{A,C,L}}{m_C \cdot L}\right)$

Since at the surface of the membrane the transport is only diffusive $\mathrm{j}_{\mathrm{A,z}} = n_A$ then:

$$\mathcal{D}_{\mathrm{AD}} \left(\frac{c_{A,B,0}}{m_B \cdot L} - \frac{c_{A,C.L}}{m_C \cdot L}\right) = k_{c,B}(c_{A,B,\,bulk} - c_{A,B,0})$$

$$\mathcal{D}_{\mathrm{AD}} \frac{c_{A,B,0}}{m_B \cdot L} - \mathcal{D}_{\mathrm{AD}} \frac{c_{A,C.L}}{m_C \cdot L} = k_{c,B}\, c_{A,B,\,bulk} - k_{c,B} c_{A,B,0}$$

$$\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right) c_{A,B,0} = k_{c,B}\, c_{A,B,\,bulk} + \mathcal{D}_{\mathrm{AD}} \frac{c_{A,C,L}}{m_C \cdot L}$$

$$c_{A,B,0} = \frac{k_{c,B}\, c_{A,B,\,bulk}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} + \frac{\mathcal{D}_{\mathrm{AD}}\, c_{A,C,L}}{m_C L \left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)}$$

Then we have

$$n_A = k_{c,B} \left(c_{A,B,\,bulk} - \frac{k_{c,B}\, c_{A,B,\,bulk}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} + \frac{\mathcal{D}_{\mathrm{AD}}\, c_{A,C,L}}{m_C L \left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} \right),$$

In this expression we can replace $c_{A,C,L}$ using $n_A = k_{c,C}(c_{A,C,L} - c_{A,C,bulk})$ or rather $c_{A,C,L} = \frac{n_A + k_{c,C} c_{A,C,bulk}}{k_{c,C}}$, Then $n_A = k_{c,B}\left(c_{A,B,\,bulk} - \frac{k_{c,B}\,c_{A,B,\,bulk}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} + \frac{\mathcal{D}_{\mathrm{AD}} \frac{n_A + k_{c,C} c_{A,C,bulk}}{k_{c,C}}}{m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} \right)$

Or $\boldsymbol{n_A} = k_{c,B}\left(c_{A,B,\,bulk} - \frac{k_{c,B}\,c_{A,B,\,bulk}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} + \mathcal{D}_{\mathrm{AD}} \frac{\boldsymbol{n_A} + k_{c,C} c_{A,C,bulk}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)} \right)$

Rearrange to solve for n_A:

$$\boldsymbol{n_A} - \frac{k_{c,B}\mathcal{D}_{\mathrm{AD}}\boldsymbol{n_A}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)} = k_{c,B} c_{A,B,\,bulk} - k_{c,B}\frac{k_{c,B}\,c_{A,B,\,bulk}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)} + \frac{k_{c,B}\mathcal{D}_{\mathrm{AD}} k_{c,C} c_{A,C,bulk}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)}$$

$$\boldsymbol{n_A}\left(1 - \frac{k_{c,B}\mathcal{D}_{\mathrm{AD}}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)}\right) = k_{c,B} c_{A,B,\,bulk}\left(1 - \frac{k_{c,B}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)}\right) + \frac{k_{c,B}\mathcal{D}_{\mathrm{AD}} k_{c,C} c_{A,C,bulk}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)}$$

$$\boldsymbol{n_A} = \frac{k_{c,B} c_{A,B,\,bulk}\left(1 - \frac{k_{c,B}}{\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B \cdot L} + k_{c,B}\right)}\right) + \frac{k_{c,B}\mathcal{D}_{\mathrm{AD}} k_{c,C} c_{A,C,bulk}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)}}{1 - \frac{k_{c,B}\mathcal{D}_{\mathrm{AD}}}{k_{c,C} m_C \cdot L\left(\frac{\mathcal{D}_{\mathrm{AD}}}{m_B . L} + k_{c,B}\right)}}$$

With some effort this can be simplified further:

$$\boldsymbol{n_A} = \frac{k_{c,B} k_{c,C} \mathcal{D}_{\mathrm{AD}}\left(m_B c_{A,C,bulk} + m_C c_{A,B,\,bulk} L^2\right)}{\mathcal{D}_{\mathrm{AD}} k_{c,C} m_C L^2 + k_{c,B} m_B (k_{c,C} m_C L - \mathcal{D}_{\mathrm{AD}})}$$

Exercise 8.4

Using the psychrometric chart, we find HR @ beginning and end

Initially, RH = 60% @ 35°C → $H_R = 0.022 \frac{g\ H_2O}{g\ dry\ air}$

After, RH = 53% @ 24°C → $H_R = 0.010 \frac{g\ H_2O}{g\ dry\ air}$

$\dot{V} = 1\ m^3/s \longrightarrow \dot{m} = 1188\ g/s$ @ 24°C

So, we must remove

$0.022 - 0.010 = 0.012\ g\ H_2O / g$ dry air

$1188\ g$ air/s $\longrightarrow 14.25\ g\ H_2O/s = 14.25\ mL\ H_2O/s = 0.86\ L/min$

$= 0.79\ mol/s$

To calculate heat removed, we use the ΔH_{vap}

$\dot{m} \cdot \Delta H_{vap} = 0.79\ mol/s \cdot 40{,}650\ J/mol = 32{,}113\ W$